

LAGRANGEAN RELAXATION WITH CLUSTERS FOR THE UNCAPACITATED FACILITY LOCATION PROBLEM

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Abstract

A good strategy for the solution of a large-scale problem is its division into small ones. In this context, this work explores the lagrangean relaxation with clusters (LagClus) that can be applied to combinatorial problems modeled by conflict graphs. By partitioning and removing the edges that connect the clusters of vertices, the conflict graph is divided in subgraphs with the same characteristics of the whole problem. When relaxing the removed edges in the lagrangean way, subproblems are solved and better limits than the traditional lagrangean relaxation are obtained. This work applies the LagClus to the Uncapacitated Facility Location Problem (UFLP).

Keywords: Lagrangean relaxation, partitioning, conflict graph, facility location.

1. INTRODUCTION

A good strategy for the solution of a large-scale problem is its division into small ones. Considering that some problems can be represented by a conflict graph that division can be taken by partitioning this graph, generating subgraphs (clusters) of vertices e edges. By removing the edges that connect the clusters, we have several subproblems totally independent and similar, in a small scale, to the original one.

The inconvenience of this approach is that there is no guarantee to get solution for the complete problem solving the subproblems, due to some edges have been removed from it. A way to consider these edges is to relax them in the lagrangean way to find a good quality limits for the original problem.

This is the idea of the lagrangean relaxation with clusters (LagClus) that considers the attainment of a conflict graph, the partitioning of this graph in clusters with the same characteristics of the original problem and the use of the lagrangean relaxation to incorporate

the removed edges to the problem resolution. Figure 1 shows the phases of this process.

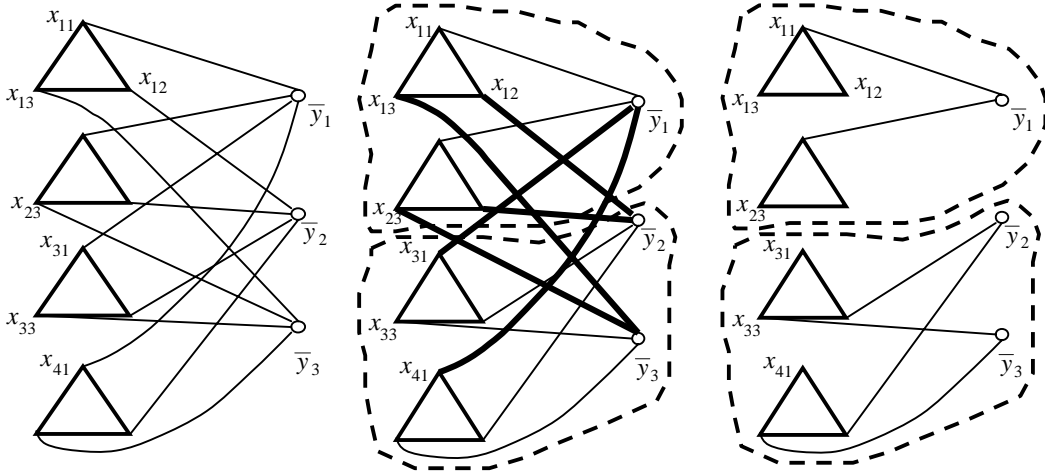


Fig.1 Description of the LagClus. (a) Conflict graph. (b) Relaxation of the removed edges. (c) Subproblems' (clusters) resolution.

The Uncapacitated Facility Location Problem (UFLP) is a location problem extensively studied in the literature that involves fixed costs for locating the facilities, production costs and transportation costs for distributing the commodities between the facilities and the clients. The main objective of the UFLP is to choose the location of facilities to minimize the cost of satisfying the demand for some commodity.

Several solution approaches have been proposed for the UFLP in the last decades. There are exact algorithms for this problem [12], but alternative algorithms (heuristics, metaheuristics and relaxations) has been a natural choice for large-scale instances, due to the NP-hard characteristic of the UFLP [4]. An effective and extensively used heuristic is the lagrangean method [3] based on the lagrangean relaxation with a subgradient optimization algorithm [6]. There are many solutions using metaheuristics, such as Simulated Annealing [1], Genetic Algorithms [13] and Tabu Search [7][14][18]. Resende & Werneck [15] have applied, to the UFLP, a hybrid multistart heuristic developed to p-median problems.

This work describes and applies the LagClus to the UFLP. The rest of this work is organized as follows. The UFLP is briefly discussed in Section 2, the LagClus is described in Section 3, computational results are reported in Section 4 and conclusions are presented in Section 5.

2. UFLP

An integer linear programming formulation for the UFLP [4] is obtained by introducing the following variables. Let $y_j = 1$ if facility j is open and $y_j = 0$ otherwise; $x_{ij} = 1$ if the demand of client i is satisfied from facility j and $x_{ij} = 0$ otherwise. We consider $i \in I$ and $j \in J$, such as I is a set of clients and J is a set of sites where facilities can be located. Let f_j be the given fixed cost of opening facility j and c_{ij} defines the cost to serve the client i from facility j . The UFLP can be formally stated as

UFLP

$$v(\text{UFLP}) = \min \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{j \in J} f_j y_j \quad (1)$$

Subject to

$$\sum_{j \in J} x_{ij} = 1 \quad \forall i \in I \quad (2)$$

$$x_{ij} \leq y_j \quad \forall i \in I, \forall j \in J \quad (3)$$

$$x_{ij}, y_j \in \{0,1\} \quad \forall i \in I, \forall j \in J \quad (4)$$

The objective is to minimize the overall system cost. The constraints (2) ensure that the demand of every client is satisfied, whereas (3) guarantees that the clients are supplied only from open facilities. The constraints (4) define the integrality requirements.

In order to compare with the LagClus, object of this article, the traditional lagrangean relaxation has also been applied to the UFLP. This approximation of the original problem was obtained by relaxing the allocation constraints (2) in the lagrangean way and incorporating them into the objective function. The L_λ UFLP problem can be stated as:

$$v(L_\lambda \text{UFLP}) = \text{Min} \sum_{i \in I} \sum_{j \in J} (c_{ij} - \lambda_i) x_{ij} + \sum_{j \in J} f_j y_j + \sum_{i \in I} \lambda_i \quad (5)$$

Subject to (3) e (4).

3. LAGRANGEAN RELAXATION WITH CLUSTERS (LAGCLUS)

The lagrangean relaxation with clusters was developed by Ribeiro [16], by adapting the results of Hicks et al. [8], which have developed a Branch-and-Price algorithm to the Maximum Weight Independent Set Problem (MWISP). MWISP belongs to the class of problems that can be decomposed in subproblems of the same type of the original one. The authors have generated a conflict graph for that problem and have partitioned it. All subgraphs (subproblems) are considered in a Branch-and-Price algorithm with subproblems generating columns for a Restricted Master Problem. Their results were good for several instances reported in the literature for the MWISP.

Ribeiro [16] e Ribeiro e Lorena [17] have report good results of LagClus for the Maximum Independent Set Problem, Point-feature Cartographic Label Problem and the Manufacturer's Pallet Loading Problem.

3.1 The LagClus process

The LagClus process can be resumed as follows:

- a) Create the conflict graph from the UFLP formulation.
- b) Apply a graph partitioning heuristic to divide the conflict graph in P clusters. The problem has constraints divided in two sets: edges connecting vertices in the same cluster and in different clusters.
- c) Relax the constraints (edges) that have vertices in different clusters.
- d) The lagrangean relaxation obtained is divided in P subproblems and solved.

3.2 Conflict graph

To get the conflict graph [2] to location problems, one have to work with the complement of the location variables ($\bar{y} = 1 - y$) [5], which applied to the UFLP becomes \overline{UFLP} .

$$v(\overline{UFLP}) = \min \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} - \sum_{j \in J} f_j \bar{y}_j + \sum_{j \in J} f_j \quad (6)$$

Subject to

$$\sum_{j \in J} x_{ij} = 1 \quad \forall i \in I \quad (7)$$

$$x_{ij} + \bar{y}_j \leq 1 \quad \forall i \in I, \forall j \in J \quad (8)$$

$$x_{ij}, \bar{y}_j \in \{0,1\} \quad \forall i \in I, \forall j \in J \quad (9)$$

In the conflict graph, as it can be observed in the example of Figure 1, the constraints (7) define cliques, where only one of the allocation variables must be equal to one in the solution. The constraints (8) define edges between allocation and location variables.

By partitioning the conflict graph $G = (V, E)$ in P parts $V_1, V_2, \dots, V_P \in \forall p \in \{1 \dots P\}$ one define subgraphs $G_p = G[V_p]$ and a edge set $E_p = E(G_p)$. The edges on G that connect the subgraphs, corresponding to vertices in different clusters, define the set $\hat{E} = E \setminus \bigcup_{p=1}^P E_p$,

Applying a graph partitioning heuristic to the conflict graph of \overline{UFLP} , a block-diagonal structure representation is obtained as follows:

$$v(\overline{UFLP}) = \min \sum_{p=1}^P \mathbf{c}^p \mathbf{x}^p - \mathbf{f}^p \bar{\mathbf{y}}^p \quad (10)$$

Subject to

$$\begin{bmatrix} A_1 & A_2 & \dots & A_p \\ B_1 & 0 & 0 & 0 \\ 0 & B_2 & 0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & B_p \end{bmatrix} \begin{pmatrix} \begin{bmatrix} \mathbf{x} \\ \bar{\mathbf{y}} \end{bmatrix}^1 \\ \begin{bmatrix} \mathbf{x} \\ \bar{\mathbf{y}} \end{bmatrix}^2 \\ \vdots \\ \begin{bmatrix} \mathbf{x} \\ \bar{\mathbf{y}} \end{bmatrix}^p \end{pmatrix} \sim \mathbf{1} \in \mathbf{x}^p, \bar{\mathbf{y}}^p \in B^{|V_p|} \quad \forall p \in \{1, \dots, P\} \quad (11)$$

where A_p is a matrix of dimensions $|\hat{E}| \times |V_p|$ of coefficients in inequalities associated with the conflict edges of \hat{E} (edges in boldface in Figure 1); B_p is a matrix of dimensions $|E| - |\hat{E}| \times |V_p|$ of coefficients in inequalities and equalities associated with the edges of E_p ; and \sim represents the relational operators $=$ or \leq depending on the respective constraint.

The subproblem $p, p \in \{1, \dots, P\}$, to the \overline{UFLP} has the form:

$$Z_p^* = \text{Min} \left\{ (\mathbf{c}^p + A_p^T \boldsymbol{\lambda}) \mathbf{x}^p - (\mathbf{f}^p - A_p^T \boldsymbol{\lambda}) \bar{\mathbf{y}}^p : \mathbf{x}^p, \bar{\mathbf{y}}^p \in Q_p \right\}$$

where $\boldsymbol{\lambda} \in R_+^K$ are the lagrangean multipliers associated with the lines of the matrix A_p ; Q_p are the constraints associated with subproblem p (cluster p).

Applying the lagrangean relaxation to this set of constraints, K being the number of relaxed edges, and solving the subproblems Z_p^* , the LagClus relaxation to the \overline{UFLP} ($LC_\lambda \overline{UFLP}$) will be:

$$v(LC_\lambda \overline{UFLP}) = \sum_{p=1}^P Z_p^* - \sum_{k=1}^K \lambda_k + \sum_{j \in J} f_j \quad (12)$$

4. COMPUTATIONAL RESULTS

The concepts presented in this work have been tested for the UFLP, using instances with large duality gap that are computationally difficult for methods based on linear relaxation [11] available at http://www.math.nsc.ru/AP/benchmarks/UFLP/Engl/uflp_dg_eng.html. Three classes of instances are presented: Gap-A, Gap-B and Gap-C. All these instances have the values $m = n = 100$ and the fixed cost to open a facility is 3000. The optimal solution value, the linear relaxation value and its duality gap have been obtained using the commercial solver CPLEX, version 7.5 [9].

These values are compared with results of the traditional lagrangean relaxation (Table 1), and with the results of the LagClus (Table 2). The subgradient heuristic has been used to optimize both lagrangean duals.

Table 1 Computational results to the linear relaxation and the lagrangean relaxation for UFLP instances with large duality gap

Problem	Optimum		Linear Relaxation			Lagrangean Relaxation		
	Opt	Time Opt (s)	LP relaxation	Duality Gap	Time LP (s)	Lag	Gap Lag	Time Lag (s)
332GapA	36154	583	26959.45	25,43	0,88	26810.08	25,84	82
432GapA	36155	1072	27223.17	24,71	0,75	27120.22	24,99	89
532GapA	36150	326	26143.15	27,68	0,83	26043.84	27,96	98
331GapB	45123	14856	34226,11	24,15	0,86	34141.59	24,34	79
431GapB	45132	18098	35031.87	22,52	0,81	34892.40	22,69	88
531GapB	45135	2287	36488.07	19,25	0,77	36432.08	19,28	133
333GapC	42147	23179	30207.90	28,33	0,86	30184.73	28,39	75
433GapC	42145	22172	30199.20	28,35	0,86	30172.98	28,42	79
533GapC	39177	3208	30200.00	22,92	0,83	30152.13	23,04	95

Table 2 Computational results obtained by the LagClus for UFLP instances with large duality gap

Problem	# clusters	LagClus	Gap LagClus	Time LagClus (s)
332GapA	2	27491,01	23,96	1913
332GapA	4	26757,59	25,99	107
432GapA	2	27872,38	22,91	2340
432GapA	4	27063,67	25,15	127
532GapA	2	26650,84	26,28	2244
532GapA	4	25998,67	28,08	155
331GapB	2	34874,42	22,71	2112
331GapB	4	34287,11	24,01	135
431GapB	2	35346,06	21,68	2010
431GapB	4	34945,41	22,57	127
531GapB	2	36891,25	18,26	1348
531GapB	4	36558,47	19,00	57
333GapC	2	31273,25	25,80	4853
333GapC	4	30078,39	28,63	328
433GapC	2	31178,82	26,02	3277
433GapC	4	30213,58	28,31	397
533GapC	2	31152,88	20,48	4232
533GapC	4	30144,18	23,06	351

The values of Gaps (Duality Gap, Gap Lag and Gap LagClus) present, in percentages, the gaps obtained between the respective relaxation and the optimal value. The times have been determined in a Pentium IV 3 GHz computer, with 1Gb of RAM memory. The LagClus subproblems have been solved with commercial solver CPLEX, version 7.5 [9].

The values in boldface of Table 2 show where the LagClus got better gaps than LP. The division of the original problem in two clusters deserves special attention presenting reduced amount of edges between them and improving the limits. For small number of clusters, we have high bound quality, with a corresponding increase in processing times.

The graph partitioning was made using the Metis software [10], a well-known heuristic for graph partitioning available at <http://www.cs.umn.edu/~metis>.

5. CONCLUSION

This work has presented the lagrangean relaxation with clusters and its application to the UFLP. The problem, represented by a conflict graph is partitioned in clusters, generating subproblems with the same characteristics of the original problem. The edges between clusters are relaxed in the lagrangean way. The lagrangean relaxation is decomposed and solved. The results show that the LagClus present dual limits with better quality than traditional relaxations for some difficult instances.

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